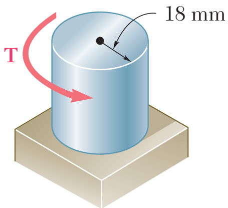


PROBLEM 3.2

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 800\ \mathrm{N \cdot m}$.

SOLUTION

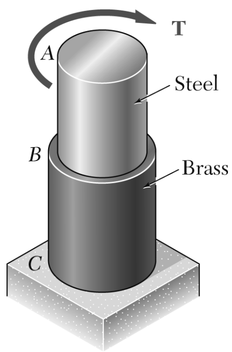
$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T}{\pi c^3}$$

$$= \frac{2(800\ \mathrm{N \cdot m})}{\pi(0.018\ \mathrm{m})^3}$$

$$= 87.328 \times 10^6\ \mathrm{Pa}$$

$\tau_{\max} = 87.3\ \mathrm{MPa}$ ◀



PROBLEM 3.15

The allowable shearing stress is 100 MPa in the 36-mm-diameter steel rod *AB* and 60 MPa in the 40-mm-diameter rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \qquad J = \frac{\pi}{2}c^4, \qquad T = \frac{\pi}{2}\tau_{\max} c^3$$

Shaft *AB*:

$$\tau_{\max} = 100 \text{ MPa} = 100 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d_{AB} = \frac{1}{2}(36) = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_{AB} = \frac{\pi}{2}(100 \times 10^6)(0.018)^3 = 916 \text{ N} \cdot \text{m}$$

Shaft *BC*:

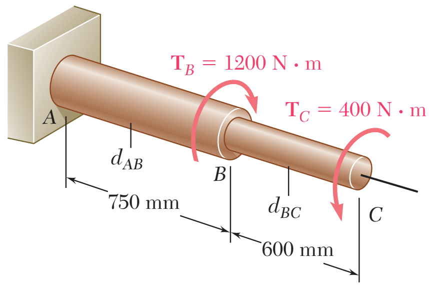
$$\tau_{\max} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d_{BC} = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

$$T_{BC} = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 = 1.754 \text{ N} \cdot \text{m}$$

The allowable torque is the smaller of T_{AB} and T_{BC}.

$T = 754 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.17

The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

SOLUTION

$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

<u>Shaft *AB*</u>:

$$T_{AB} = 1200 - 400 = 800 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55 \times 10^6)}} = 21.00 \times 10^{-3} \text{ m} = 21.0 \text{ m}$$

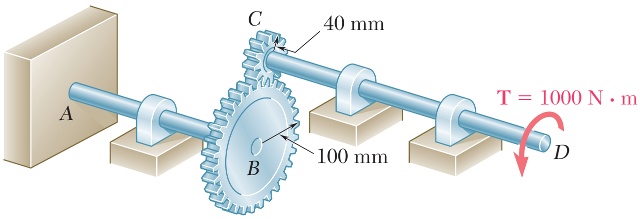
minimum $d_{AB} = 2c = 42.0$ mm ◀

<u>Shaft *BC*</u>:

$$T_{BC} = 400 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◀



PROBLEM 3.21

A torque of magnitude $T = 1000\ \text{N} \cdot \text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.

SOLUTION

$$T_{CD} = 1000\ \text{N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N} \cdot \text{m}$$

(*a*) Shaft *AB*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}\ \text{m}^3$$

$$c = 29.82 \times 10^{-3} = 29.82\ \text{mm}$$

$d = 2c = 59.6\ \text{mm}$ ◀

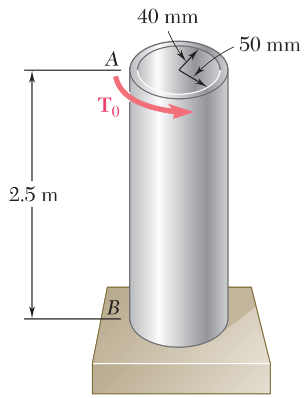
(*b*) Shaft *CD*:

$$\tau_{\text{all}} = 60 \times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1000)}{\pi(60 \times 10^6)} = 10.610 \times 10^{-6}\ \text{m}^3$$

$$c = 21.97 \times 10^{-3}\ \text{m} = 21.97\ \text{mm}$$

$d = 2c = 43.9\ \text{mm}$ ◀



PROBLEM 3.34

(a) For the aluminum pipe shown ($G = 27$ GPa), determine the torque $\mathbf{T}_0$ causing an angle of twist of 2°. (b) Determine the angle of twist if the same torque $\mathbf{T}_0$ is applied to a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(a)

$$c_o = 50 \text{ mm} = 0.050 \text{ m}, \quad c_i = 40 \text{ mm} = 0.040 \text{ m}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(0.050^4 - 0.040^4)$$

$$= 5.7962 \times 10^{-6} \text{ m}^4$$

$$\varphi = 2° = 34.907 \times 10^{-3} \text{ rad} \qquad L = 2.5 \text{ m}$$

$$G = 27 \times 10^9 \text{ Pa}$$

$$\varphi = \frac{TL}{GJ} \qquad T_0 = \frac{GJ\varphi}{L} = \frac{(27 \times 10^9)(5.7962 \times 10^{-6})(34.907 \times 10^{-3})}{2.5}$$

$$= 2.1851 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_0 = 2.19 \text{ kN} \cdot \text{m} \blacktriangleleft$$

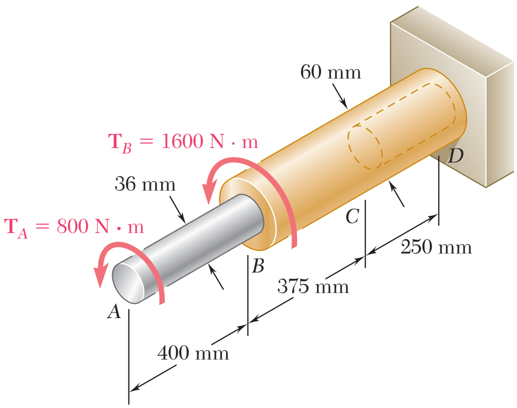
Area of pipe: $A = \pi(c_o^2 - c_i^2) = \pi(0.050^2 - 0.040^2) = 2.8274 \text{ m}^2$

(b) Radius of solid of same area: $c_s = \sqrt{\frac{A}{\pi}} = 0.030 \text{ m}$

$$J_s = \frac{\pi}{2}c_s^4 = \frac{\pi}{2}(0.030)^4 = 1.27235 \times 10^{-6} \text{ m}^2$$

$$\varphi_s = \frac{T_0 L}{GJ} = \frac{(2.1851 \times 10^3)(2.5)}{(27 \times 10^9)(1.27235 \times 10^{-6})} = 0.15902 \text{ rad}$$

$$\varphi_s = 9.11° \blacktriangleleft$$



PROBLEM 3.38

The aluminum rod *AB* (G = 27 GPa) is bonded to the brass rod *BD* (G = 39 GPa). Knowing that portion *CD* of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at *A*.

SOLUTION

Rod *AB*:

$$G = 27 \times 10^9 \text{ Pa}, \quad L = 0.400 \text{ m}$$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part *BC*:

$$G = 39 \times 10^9 \text{ Pa} \quad L = 0.375 \text{ m}, \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part *CD*:

$$c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad L = 0.250 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

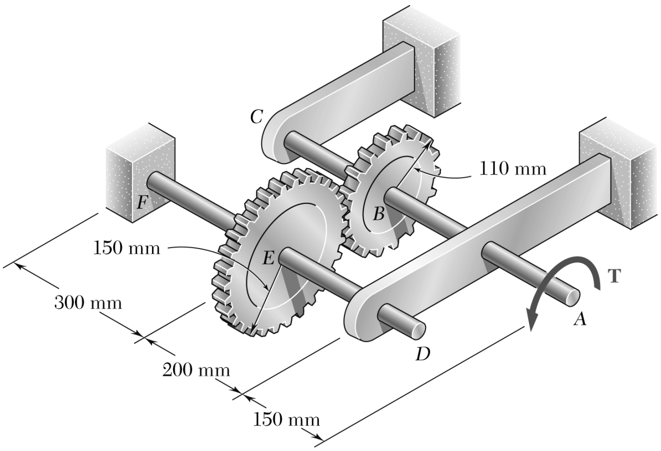
$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at *A*.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$

$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\varphi_A = 6.02° \blacktriangleleft$$



PROBLEM 3.41

Two shafts, each of 22-mm diameter are connected by the gears shown. Knowing that $G = 77$ GPa and that the shaft at F is fixed, determine the angle through which end A rotates when a 130 N · m torque is applied at A.

SOLUTION

Calculation of torques.

Circumferential contact force between gears B and E. $\quad F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E} \quad T_{EF} = \frac{r_E}{r_B} T_{AB}$

$$T_{AB} = 130 \text{ N} \cdot \text{m}$$

$$T_{EF} = \frac{150}{110}(130) = 177.3 \text{ N} \cdot \text{m}$$

Twist in shaft FE.

$$L = 300 \text{ mm}, \quad c = \frac{1}{2}d = 11 \text{ mm}, \quad G = 776 \text{ Pa}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.011)^4 = 23 \times 10^{-9} \text{ m}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(177.3)(0.3)}{(77 \times 10^9)(23 \times 10^{-9})} = 0.03 \text{ rad}$$

Rotation at E. $\quad \varphi_E = \varphi_{E/F} = 0.03 \text{ rad}$

Tangential displacement at gear circle $\quad \delta = r_E \varphi_E = r_B \varphi_B$

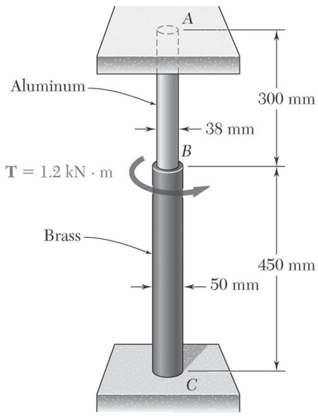
Rotation at B. $\quad \varphi_B = \frac{r_E}{r_B}\varphi_E = \frac{15}{11}(0.03) = 0.0409 \text{ rad}$

Twist in shaft BA. $\quad L = 0.35 \text{ m} \quad J = 23 \times 10^{-9} \text{ m}^4$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(130)(0.35)}{(77 \times 10^9)(23 \times 10^{-9})} = 0.0257 \text{ rad}$$

Rotation at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 0.0666 \text{ rad} \qquad \varphi_A = 3.82° ◀$

Problem 3.51



3.51 The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 26 GPa for aluminum and 39 GPa for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

The torques in cylinders AB and BC are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder AB $\quad c = \frac{1}{2}d = 0.019\text{ m} \quad L = 0.3\text{ m}$

$$J = \frac{\pi}{2}c^4 = 204.7 \times 10^{-9}\text{ m}^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(0.3)}{(26\times10^9)(204.7\times10^{-9})} = 56.37\times10^{-6}\,T_{AB}$$

Cylinder BC: $\quad c = \frac{1}{2}d = 0.025\text{ m} \quad L = 0.45\text{ m}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.6\times10^{-9}\text{ m}^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(0.45)}{(39\times10^9)(613.6\times10^{-9})} = 18.8\times10^{-6}\,T_{BC}$$

Matching expressions for φ_B. $\quad 56.37\times10^{-6}\,T_{AB} = 18.8\times10^{-6}\,T_{BC}$

$$T_{BC} = 2.9984\,T_{AB} \qquad (1)$$

Equilibrium of connection at B: $\quad T_{AB} + T_{BC} - T = 0 \quad T = 1200\text{ Nm}$

$$T_{AB} + T_{BC} = 1.2\times10^3 \qquad (2)$$

Substituting (1) into (2), $\quad 3.9984\,T_{AB} = 1.2\times10^3$

$T_{AB} = 300\text{ Nm} \qquad T_{BC} = 900\text{ Nm}$

(a) Maximum stress in cylinder AB.

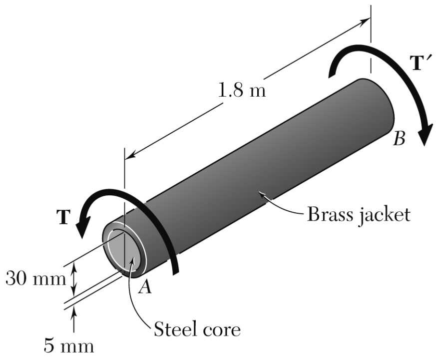
$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(300)(0.019)}{204.7\times10^{-9}} = 27.85\text{ MPa}$$

$\tau_{AB} = 27.9\text{ MPa}$ ◀

(b) Maximum stress in cylinder BC.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(900)(0.025)}{613.6\times10^{-9}} = 36.67\text{ MPa}$$

$\tau_{BC} = 36.7\text{ MPa}$ ◀



PROBLEM 3.53

The composite shaft shown consists of a 5 mm thick brass jacket ($G = 39$ GPa) bonded to a 30 mm diameter steel core ($G = 77$ GPa). Knowing that the shaft is subjected to 565 N · m torques, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of end *B* relative to end *A*.

SOLUTION

Steel core:

$$c_1 = \frac{1}{2}d = 15 \text{ mm}$$

$$J_1 = \frac{\pi}{2}c_1^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9} \text{m}^4$$

$$G_1 J_1 = (77 \times 10^9)(79.52 \times 10^{-9}) = 6123.04 \text{ N} \cdot \text{m}^2$$

Torque carried by steel core

$$T_1 = G_1 J_1 \frac{\varphi}{L}$$

Brass jacket:

$$c_2 = c_1 + t = 15 \text{ mm} + 5 \text{ mm} = 20 \text{ mm}$$

$$J_2 = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.02^4 - 0.015^4) = 0.1718 \times 10^{-6} \text{m}^4$$

$$G_2 J_2 = (39 \times 10^9)(0.1718 \times 10^{-6}) = 6700.4 \text{ N} \cdot \text{m}^2$$

Torque carried by brass jacket

$$T_2 = G_2 J_2 \frac{\varphi}{L}$$

Total torque

$$T = T_1 + T_2 = (G_1 J_1 + G_2 J_2)\frac{\varphi}{L}$$

$$\frac{\varphi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{565}{6123.04 + 6700.4} = 0.044 \text{ rad/m}$$

(*a*) Maximum shearing stress in brass jacket

$$\tau_{\max} = G_2 \gamma_{\max} = G_2 C_2 \frac{\varphi}{L} = (39 \times 10^9)(0.02)(0.044)$$

$$= 34.37 \times 10^6 \text{ N/m}^2$$

34.37 MPa ◀

(*b*) Maximum shearing stress in steel core

$$\tau_{\max} = G_1 \gamma_{\max} = G_1 C_1 \frac{\varphi}{L} = (77 \times 10^9)(0.015)(0.044)$$

$$= 50.82 \times 10^6 \text{ N/m}^2$$

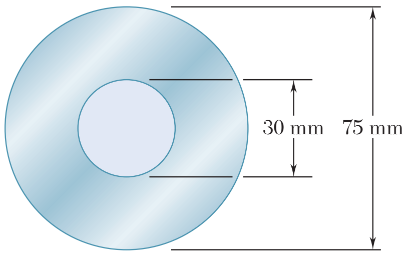
50.82 MPa ◀

(c) Angle of twist ($L = 1.8$ m)

$$\varphi = L\frac{\varphi}{L} = (1.8)(0.044) = 79.2 \times 10^{-3} \text{ rad}$$

$$= \mathbf{4.53°}$$

◀



PROBLEM 3.68

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 4-m length. Using $G = 77.2$ GPa, determine the power being transmitted.

SOLUTION

Twist angle: $\varphi = 2° = 34.907 \times 10^{-3}$ rad

$$c_1 = \frac{1}{2}d_1 = 0.015 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.0375 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0375^4 - 0.015^4)$$

$$J = 3.0268 \times 10^{-6} \text{ m}^4, \qquad L = 4 \text{ m}$$

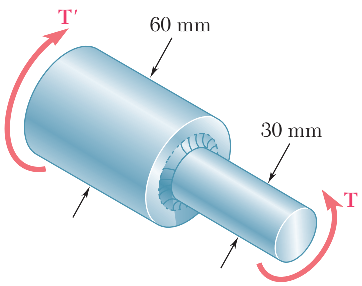
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(3.0268 \times 10^{-6})(34.907 \times 10^{-3})}{4}$$

$$T = 2.0392 \times 10^3 \text{ N} \cdot \text{m}$$

$$f = 120 \text{ rpm} = \frac{120}{60} \text{ Hz} = 2 \text{ Hz}$$

$$P = (2\pi f)T = 2\pi(2)(2.0392 \times 10^3) = 25.6 \times 10^3 \text{ W} = 25.6 \text{ kW}$$ ◀

PROBLEM 3.87



The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \qquad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \quad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

From Fig. 3.32, $K = 1.18$

Allowable torque. $$T = \frac{\pi(15 \times 10^{-3})^3(45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$$

Maximum power. $$P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$

$P = 63.5$ kW ◀